A theoretical analysis on the sensitivity of microwave emission to snow parameters

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Outline

- Motivations
- The bistatic AIEM model and its validation
- The DMRT-QCA
- Validation of the IRIDE model
- Sensitivity analysis
- Conclusions

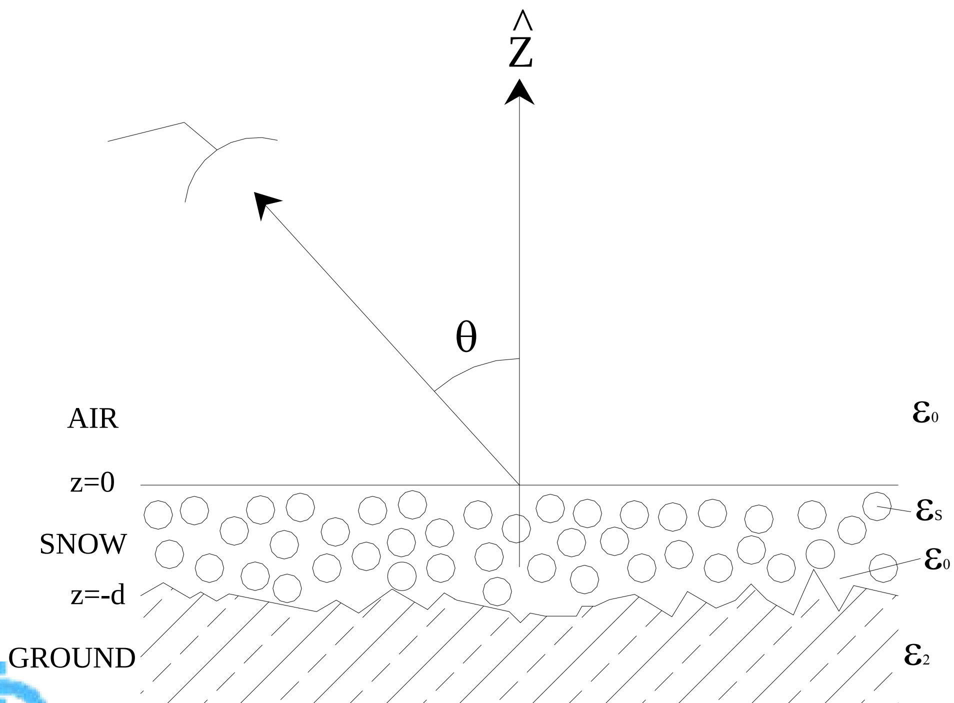
Motivations

- In literature, the analysis of the sensitivity of brightness temperature to snow parameters is scattered here and there in several journals and books published over a time-span of more than twenty years.
- Some analyses are made by means of experimental data, other with numerical simulations but it is hard to compare them because the snow parameters are quite different
- A systematic analysis is useful in view of retrieval algorithm developments

The model

•This work aims at systematically investigating the sensitivity of Tb to the snow parameters by means of an advanced model called IRIDE (Ifac RadIative Dry Snow Emission model)

To this end we coupled surface and volume scattering models



➢Snow is modeled as a layer of sticky spherical scatterers

➢The air-snow interface is considered flat

➢The soil roughness is accounted for by means of the AIEM

The AIEM model

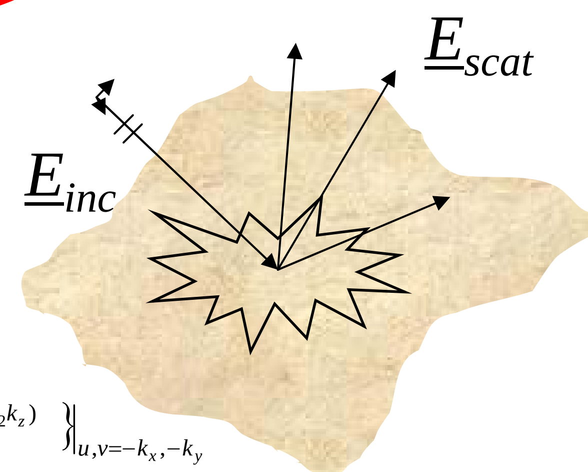
The bistatic scattering coefficient is composed of three terms: Kirchhoff, cross and the complementary one.

$$\sigma^{o}{}_{qp}(S) = \sigma^{k}{}_{qp}(S) + \sigma^{kc}{}_{qp}(S) + \sigma^{c}{}_{qp}(S) =$$

$$= \frac{k_1^{\,2}}{2} e^{-\sigma^2 (k_z^2 + k_{sz}^2)} \cdot \sum_{n=1}^{\infty} \frac{\sigma^{2n}}{n!} \left| I_{qp}^{n} \right|^2 W^{(n)}(k_{sx} - k_x, k_{sy} - k_y)$$

spectrum

Intensity terms



$$\begin{aligned}
I_{qp}^{n} = \; & (k_z + k_{sz})^n \; f_{qp} \; e^{-\sigma^2 k_z k_{sz}} \\
& + \frac{1}{4} \Big\{ (k_{sz} - q_1)^n \; F_{qp1}^{(+)} \; e^{-\sigma^2 (q_1^2 - q_1 k_{sz} + q_1 k_z)} + (k_{sz} - q_2)^n \; F_{qp2}^{(+)} \; e^{-\sigma^2 (q_2^2 - q_2 k_{sz} + q_2 k_z)} \\
& \quad + (k_{sz} + q_1)^n \; F_{qp1}^{(-)} \; e^{-\sigma^2 (q_1^2 + q_1 k_{sz} - q_1 k_z)} + (k_{sz} + q_2)^n \; F_{qp2}^{(-)} \; e^{-\sigma^2 (q_2^2 + q_2 k_{sz} - q_2 k_z)} \Big\} \Big|_{u,v = -k_x, -k_y} \\
& + \frac{1}{4} \Big\{ (k_z + q_1)^n \; F_{qp1}^{(+)} \; e^{-\sigma^2 (q_1^2 - q_1 k_{sz} + q_1 k_z)} + (k_z + q_2)^n \; F_{qp2}^{(+)} \; e^{-\sigma^2 (q_2^2 - q_2 k_{sz} + q_2 k_z)} \\
& \quad + (k_z - q_1)^n \; F_{qp1}^{(-)} \; e^{-\sigma^2 (q_1^2 + q_1 k_{sz} - q_1 k_z)} + (k_z - q_2)^n \; F_{qp2}^{(-)} \; e^{-\sigma^2 (q_2^2 + q_2 k_{sz} - q_2 k_z)} \Big\} \Big|_{u,v = -k_{sx}, -k_{sy}}
\end{aligned}$$

Model assumptions

•The reflection coefficients are computed by means of a threshold criterion similar to (Fung, 1994).

$$R_{v,h} = \begin{cases} R(\theta_i) & k^2\sigma L < 1.2\sqrt{\varepsilon_r} \\ R(\theta_{spec}) & k^2\sigma L > 1.2\sqrt{\varepsilon_r} \end{cases}$$

•The shadowing effect is accounted for as in (Tsang and Kong, 2001).

$$S(\hat{k}_s, \hat{k}_i) = \frac{1}{\Lambda(\theta_s) + \Lambda(\theta_i) + 1} \qquad \Lambda(\mu) = \frac{1}{2}\left[\sqrt{\frac{2}{\pi}}\frac{s}{\cot(\mu)} e^{-\frac{\cot(\mu)^2}{2s^2}} - erfc\left(\frac{\cot(\mu)}{s\sqrt{2}}\right)\right]$$

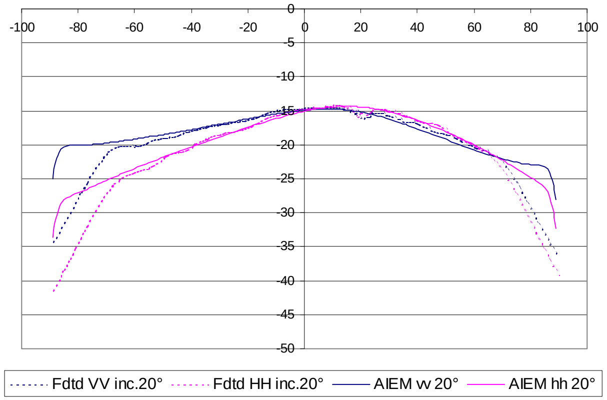
•Six autocorrelation functions have been considered. The 1.5-power ACF is used.

$$W^{(n)}(k_x, k_y) = L^2\left(\frac{KL}{2}\right)^{1.5n+1} \frac{\mathrm{K}_{-(1.5n+1)}\left([k_x{}^2 + k_y{}^2]L^2\right)}{\Gamma(1.5n)}$$

AIEM validation (bistatic)

The coded AIEM has been validated over a wide range of surfaces

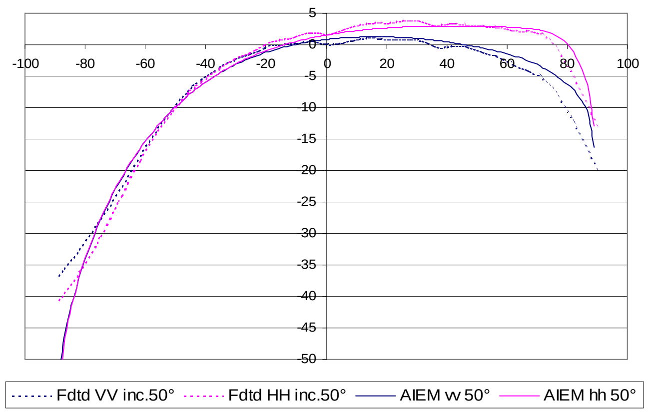
Numerical dataset (fdtd)



ks=0.12
kl=1.48
ε=8.5+j1.2

θ = - 20°

exponential



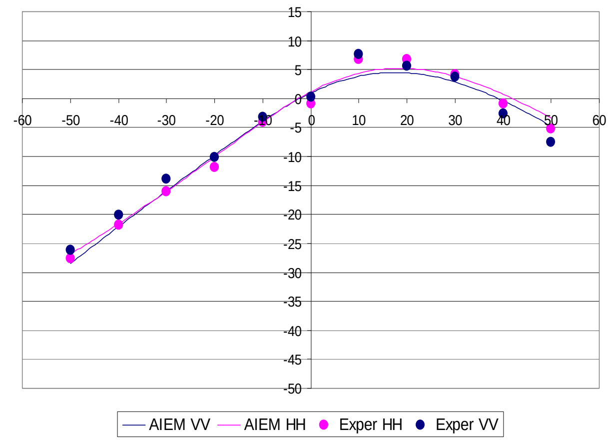
ks=3.5
kl=12.56

ε= 24.6 + j 3.19

θ = - 50°

gaussian

Experimental dataset (EMSL-JRC)



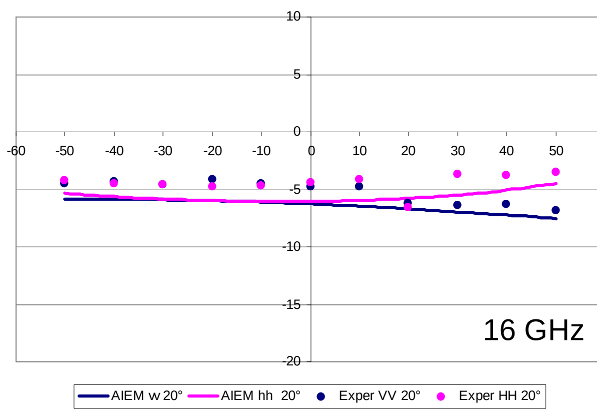
C- band

ε= 8.7+ j 2.8

S=0.4cm
L=6cm

θ_i = -20°

gaussian



Ku-band

ε= 5.5 – j 2.2

S=2.5cm
L=6cm

θ_i = - 20°

gaussian

The snow model

The electromagnetic properties of the snow are computed according to the DMRT-QCA (Tsang et al., 2007)

$$\cos\theta \frac{d\vec{I}(\theta,\phi,z)}{dz} = -\overline{\overline{k_e}}(\theta,\phi)\vec{I}(\theta,\phi,z) + k_a C \vec{T}_1 + \int_0^{\pi/2} d\theta' \sin\theta' \int_0^{2\pi} d\phi' [\overline{\overline{P}}(\theta,\phi;\theta',\phi')\vec{I}(\theta',\phi',z) + \overline{\overline{P}}(\theta,\phi;\pi-\theta',\phi')\vec{I}(\pi-\theta',\phi',z)]$$

$$\begin{bmatrix} I_{1s} \\ I_{2s} \\ U_{12s} \\ V_{12s} \end{bmatrix} = \begin{bmatrix} P_{11}(\Theta) & 0 & 0 & 0 \\ 0 & P_{22}(\Theta) & 0 & 0 \\ 0 & 0 & P_{33}(\Theta) & P_{34}(\Theta) \\ 0 & 0 & P_{43}(\Theta) & P_{44}(\Theta) \end{bmatrix} \cdot \begin{bmatrix} I_{1i} \\ I_{2i} \\ U_{12i} \\ V_{12i} \end{bmatrix}$$

$$P_{11}(\Theta) = |f_{11}(\Theta)|^2 q(\Theta)$$

$$P_{22}(\Theta) = |f_{22}(\Theta)|^2 q(\Theta)$$

$$P_{33}(\Theta) = \mathrm{Re}\left(f_{11}(\Theta) f_{22}^*(\Theta)\right) q(\Theta)$$

$$P_{34}(\Theta) = -\mathrm{Im}\left(f_{11}(\Theta) f_{22}^*(\Theta)\right) q(\Theta)$$

$$P_{33}(\Theta) = P_{44}(\Theta)$$

$$P_{43}(\Theta) = -P_{34}(\Theta)$$

$$f_{11}(\Theta) = -\frac{i}{(1-R)}\sqrt{\frac{1}{k\,K_r}} \sum_{n=1}^{N_{\max}} \frac{2n+1}{n(n+1)} \left[T_n^{(M)} X_n^{(M)} \tau_n(\cos\Theta) + T_n^{(N)} X_n^{(N)} \pi_n(\cos\Theta)\right]$$

$$f_{22}(\Theta) = -\frac{i}{(1-R)}\sqrt{\frac{1}{k\,K_r}} \sum_{n=1}^{N_{\max}} \frac{2n+1}{n(n+1)} \left[T_n^{(M)} X_n^{(M)} \pi_n(\cos\Theta) + T_n^{(N)} X_n^{(N)} \tau_n(\cos\Theta)\right]$$

$$k_s = \pi \int_0^{\pi} \left(P_{11}(\Theta) + P_{22}(\Theta)\right) \sin\Theta \; d\Theta$$

$$k_a = \frac{k}{K_r} \frac{2\pi}{k^2 |1-R|^2} n_0 \sum_{n=1}^{N_{\max}} (2n+1) \left[\left|X_n^{(M)}\right|^2 + \left(-\mathrm{Re}\, T_n^{(M)} - \left|T_n^{(M)}\right|^2\right) + \left|X_n^{(N)}\right|^2 + \left(-\mathrm{Re}\, T_n^{(N)} - \left|T_n^{(N)}\right|^2\right) \right]$$

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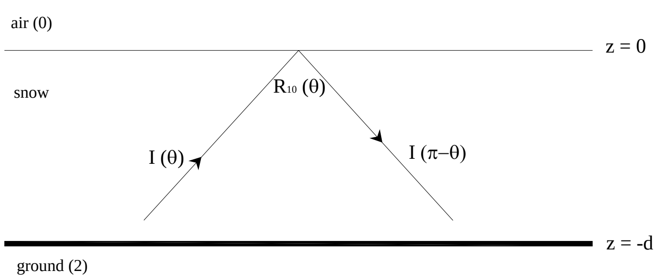
DMRT-QCA

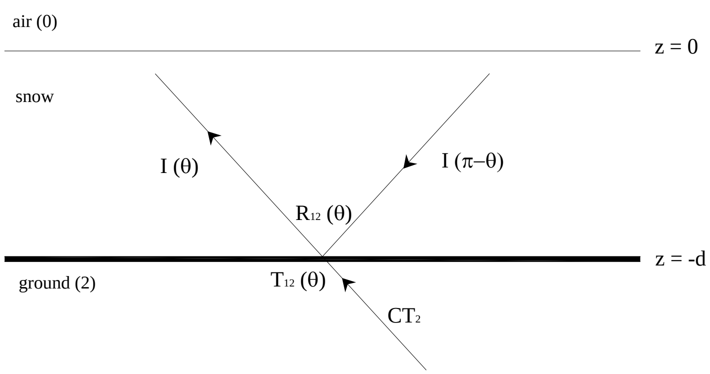
The radiative transfer equation has been solved by means of the discrete ordinate eigenanalysis technique.

$$\cos\theta \frac{d\vec{I}(\theta,\phi,z)}{dz} = -\overline{\overline{k_e}}(\theta,\phi)\vec{I}(\theta,\phi,z) + k_a C\vec{T}_1 +$$

$$+\int_0^{\pi/2} d\theta' \sin\theta' \int_0^{2\pi} d\phi' \cdot [\overline{\overline{P}}(\theta,\phi;\theta',\phi')\vec{I}(\theta',\phi',z) + \overline{\overline{P}}(\theta,\phi;\pi-\theta',\phi')\vec{I}(\pi-\theta',\phi',z)]$$

- *P* and I have been expanded into Fourier series to eliminate the dependence from ϕ
- The integral has been approximated by Gauss-Legendre formula
- A differential equation is obtained with a source term $k_a C T_1$
- The homogeneous solution is solved by imposing the boundary conditions





- The particular solution is added

DMRT-QCA

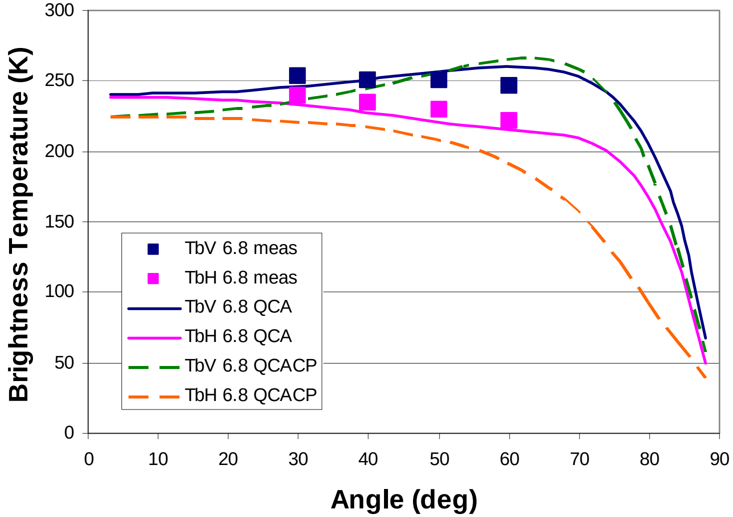
The equivalent reflectivity of the lower rough interface (soil) is obtained by computing the integral of the γ_{rr} and γ_{tr} predicted by AIEM

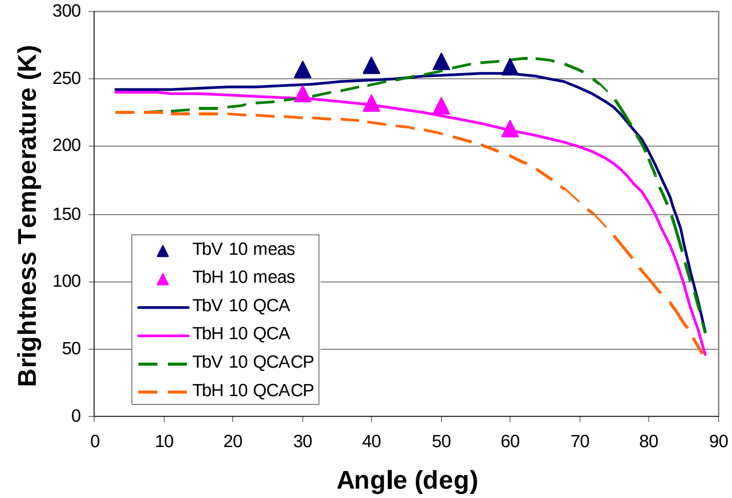
$$\Gamma_r(\theta) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \left[\gamma_{rr}(\theta_s, \theta \ , \phi_s - \phi \ ) + \gamma_{tr}(\theta_s, \theta \ , \phi_s - \phi \ ) \right] \sin\theta_s \ d\theta_s d\phi_s - \phi$$

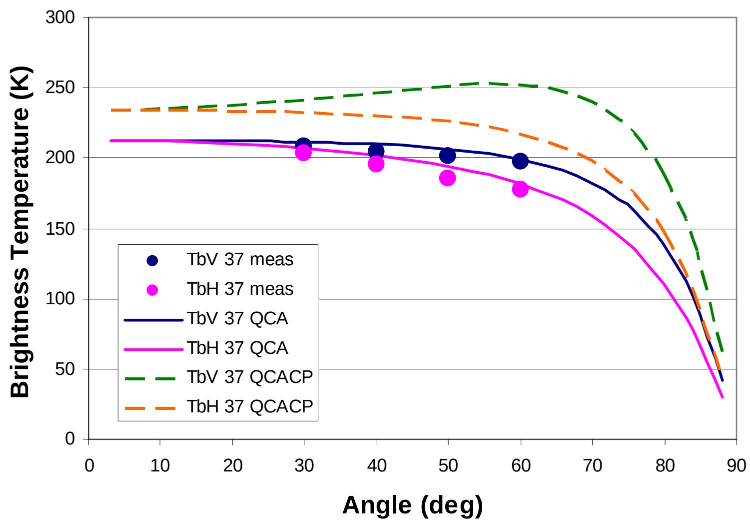
$$t, r = v, h$$

IRIDE validation

Data taken on the Eastern Italian Alps: Campolongo test site



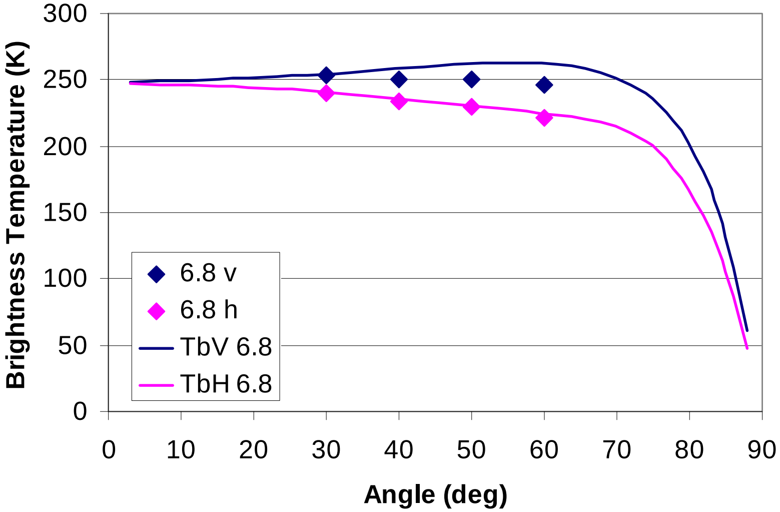


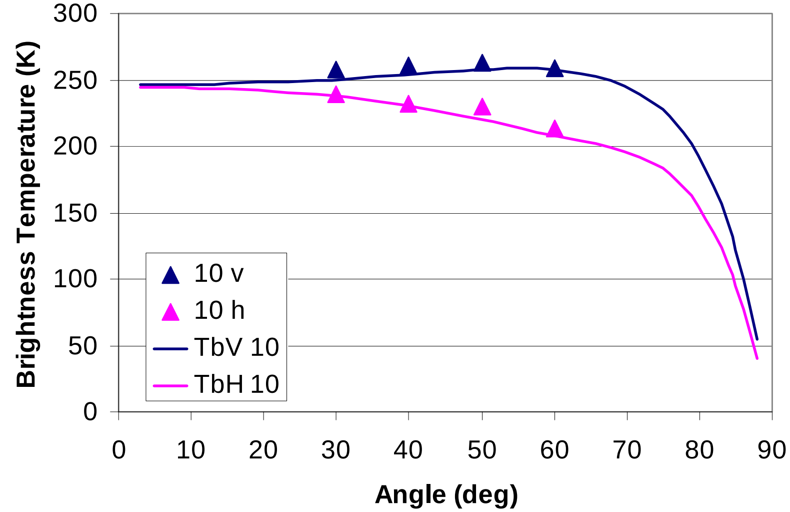


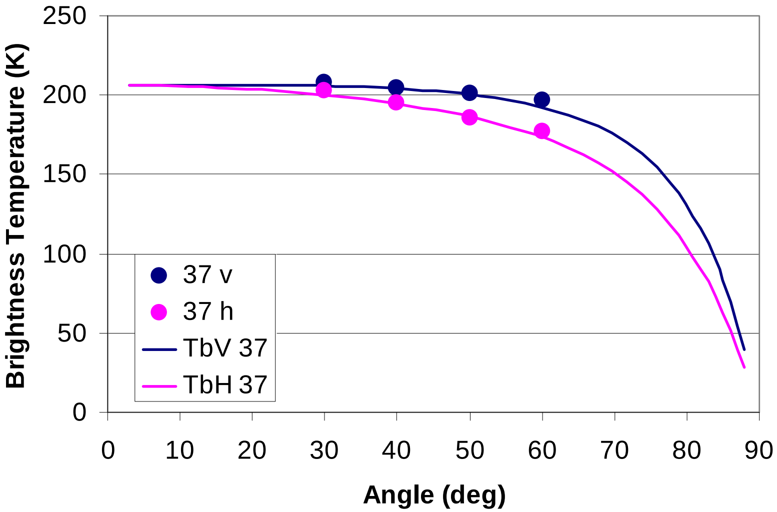
Snow depth = 0.67m
Frac vol = 0.3
Grain size = 1mm (6.8, 10 GHz),
0.6mm (37GHz)
Tsnow = 266 K
Tsoil = 271 K
Epsilon soil = 8+j2
HStdD = 0.5 cm
Correl. Len. = 20 cm
ACF = 1.5 power

IRIDE validation

Data taken on the Eastern Italian Alps: Zingari Alti test site

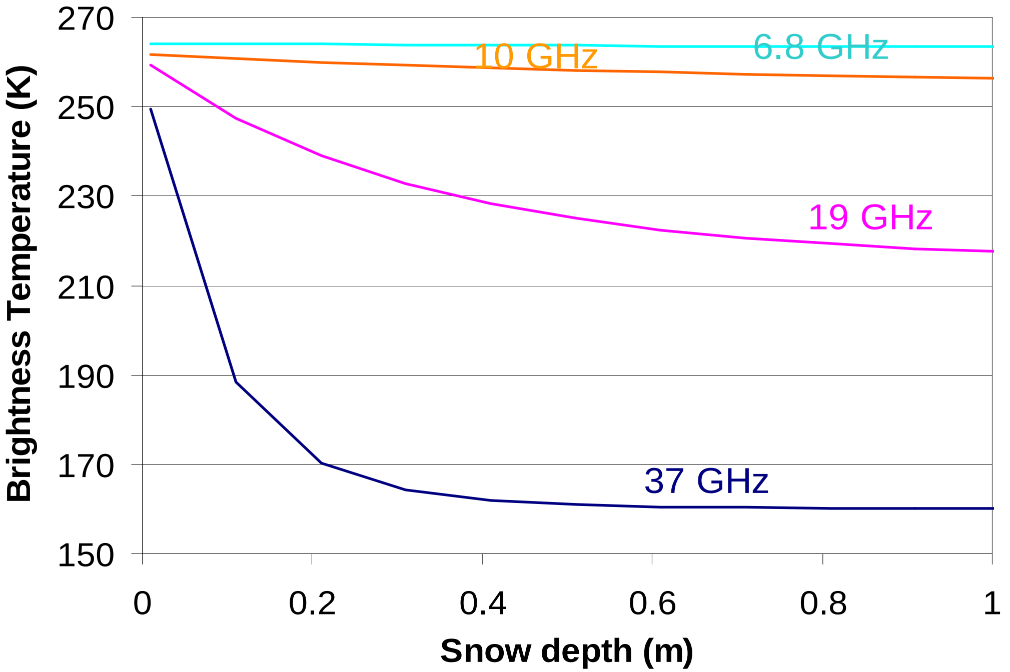






Snow	
Temperature (K)	266
Depth (m)	1.16
Fractional volume	0.3
Particle diameter (mm)	0.7 @ 6.8,10 GHz 0.4 @ 37 GHz
Stickyness	0.1
Soil	
Permittivity	5+j1
HSdtD (cm)	0.5
Correlation length (cm)	12
ACF	1.5 power

Sensitivity to Snow Depth



θ= 55°

Vpol

Fraz. Vol.= 0.3

Diam. cristalli = 1 mm

sds = 0.5 cm

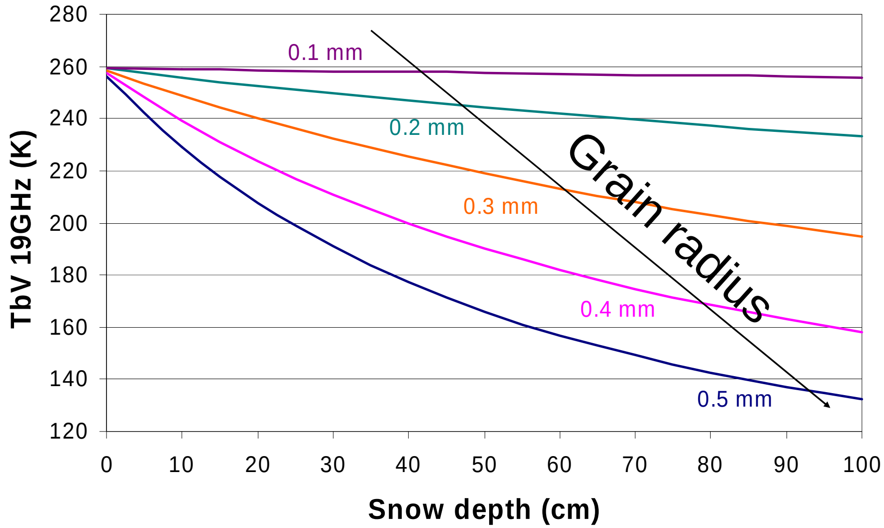
Lungh. correlaz. = 20 cm

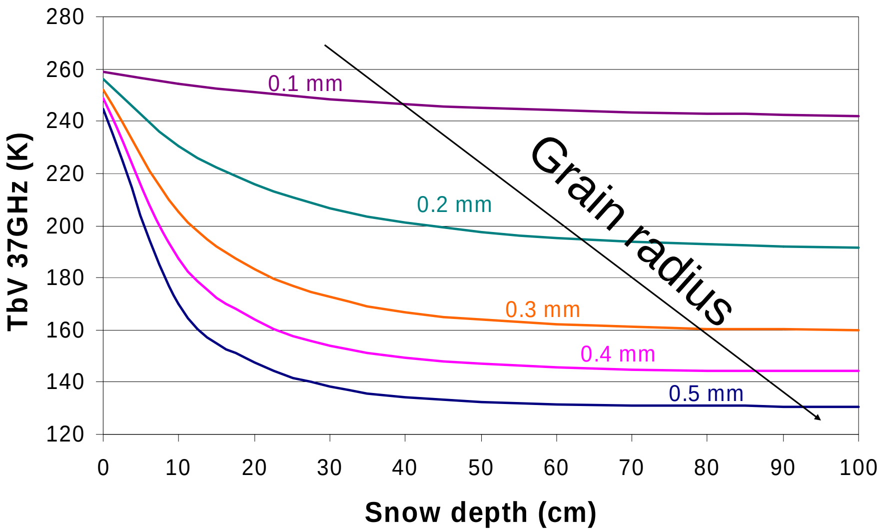
ACF=1.5 power

epst=8+j2

- At the lowest frequencies, the emission depends mainly on soil.
- At Ku and Ka bands the extinction properties of the ice crystals become appreciable and brightness decreases as SWE increases.

Sensitivity to Snow Depth





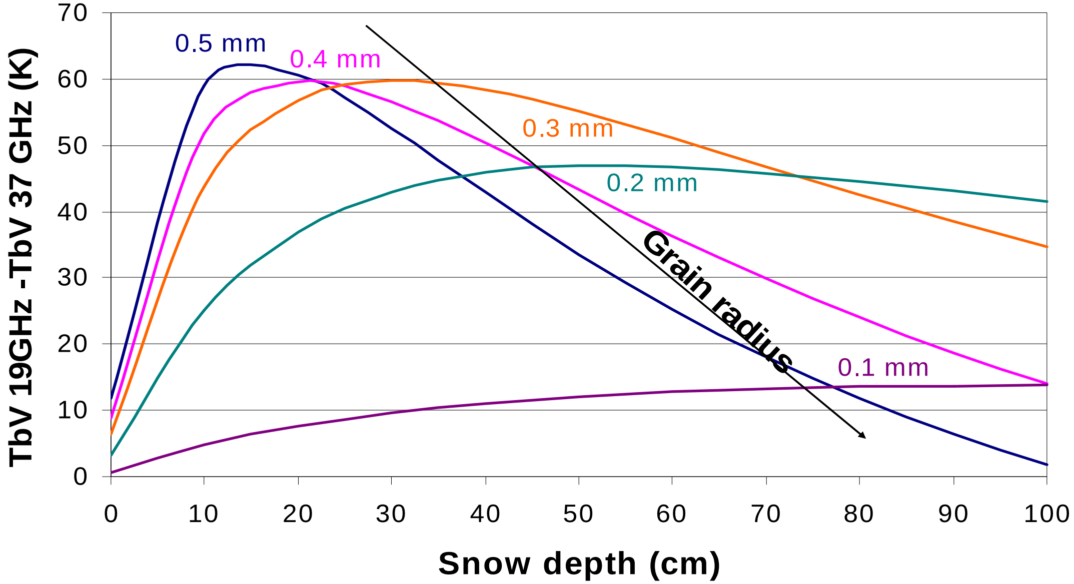
- 19 GHz: good sensitivity (nearly linear) of Tb to SD for almost all particle sizes.

- 37 GHz: the sensitivity is higher with saturation at relatively low SD. For shallow snowpacks the sensitivity is almost constant

(The soil and the snow parameters are set to typical values of Alpine snow covers)

parameter	*value*
Fractional volume	0.2
Grain radius	0.1-0.5mm
Scatterers permittivity	3.2+j0.002 @ 19 GHz 3.2+j0.01 @ 37 GHz
Stickiness τ	0.1
Snow temperature	260 K
Soil temperature	260 K
Ground permittivity	3.2

Sensitivity of the Chang's algorithm



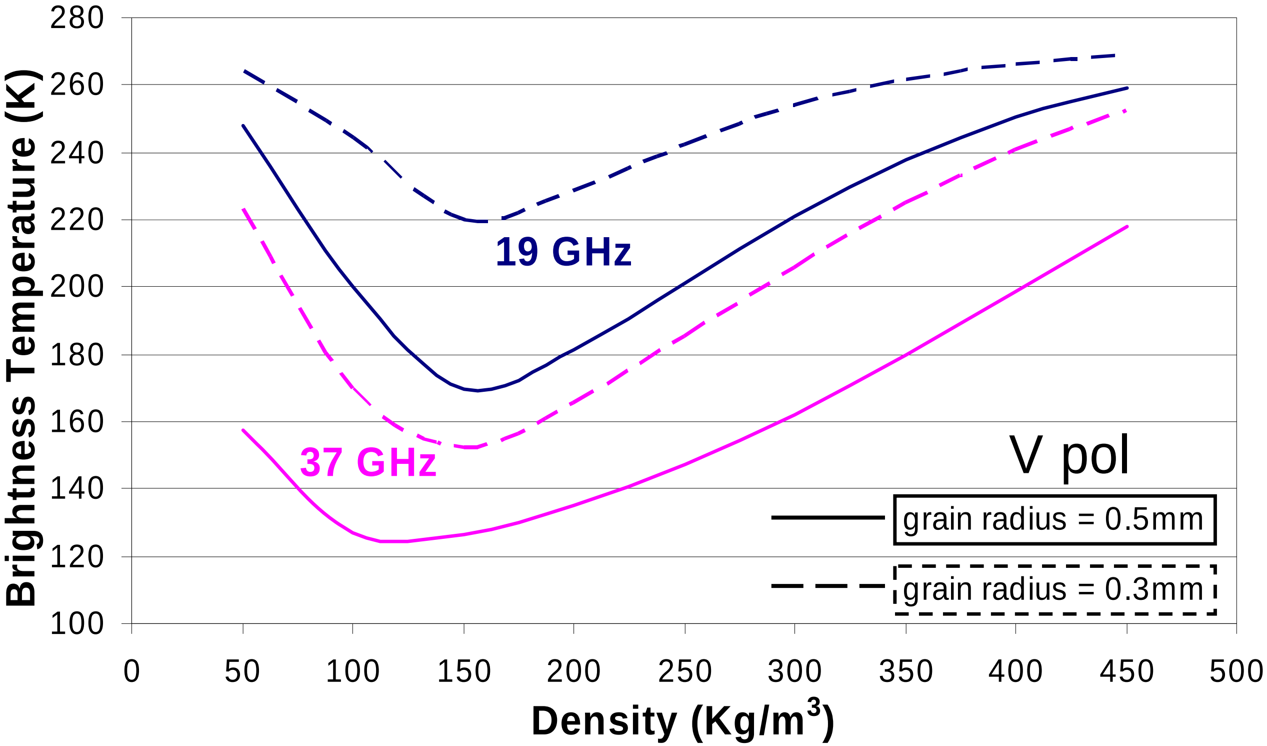
$$Snow\ Depth = a\left(T_{19H/V} - T_{37H/V}\right) + b$$

☞ For small grain size ΔTb = Tb(19)-Tb(37) increases almost linearly up to a depth of 1m.

☞ As the grain radius increases Δ Tb presents a maximum and the algorithm can fail. This is due to the saturation showed by the emission at 37 GHz.

Empirical algorithms can give acceptable predictions only if applied to large areas, where the large variability of the grain size can smooth the sharp transition that occurs when the crystal radius is bigger than 0.2 mm

Sensitivity to snow density



parameter	*value*
Frequency	19,37 GHz
Observation angle	55 deg
Density	0.05-0.45 Kg/m^3
Grain radius	0.3, 0.5 mm
Scatterers permittivity	Hufford model
Snow depth	1 m
Stickiness τ	0.1
Snow temperature	266 K
Soil temperature	271 K
Ground permittivity	8+j2

Tb is affected in opposite ways by absorption and scattering. The trend is the same for the four cases: for very tenuous snow, scattering is predominant and Tb decreases almost linearly, after a minimum is reached (for density ~150 Kg/m3) the absorption/emission prevails and Tb starts to rise.

Conclusion and future works

- ✓ The IRIDE model has been developed by coupling two models at the state of the art for both surface and volume scattering.
- ✓ The AIEM has been coded and validated against numerical and experimental datasets in the framework of an ESA project.
- ✓ The last version of the DMRT-QCA has been verified and implemented.
- ✓ A sensitivity analysis has been carried out with respect the most important parameters for snow radiometry: frequency, density, snow depth, grain size.
- ✓ As expected ,at the lowest frequencies (C- and X-band) the snow is almost transparent to microwaves
- ✓ The sensitivity to snow density is not monotonic and presents a minimum in between 150 and 250 Kg/m^3

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Conclusion and future works

- The soil roughness characterization needs to be improved
- Possible improvements can be obtained by considering the vertical profile of the snow parameters. Such task can be accomplished by solving the radiative transfer equation with the matrix doubling technique.

Thanks for the attention

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